

Thermal and Quantum Superstring Cosmologies*

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Abstract

We consider four dimensional heterotic string backgrounds for which supersymmetry is spontaneously broken via the Scherk-Schwarz mechanism on an internal spatial cycle and by finite temperature effects. We concentrate on initially flat backgrounds with $N = 4$ and $N = 2$ amount of supersymmetry. Thermal and quantum corrections give rise to a non-trivial cosmological evolution. We show that these corrections are under control and calculable due to the underlying no-scale structure of the effective supergravity theory. The effective Friedmann-Hubble equation involves a radiation term $\sim 1/a^4$ and a curvature term $\sim 1/a^2$, whose coefficients are functions of ratio of the gravitino mass scale to the temperature.

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1 Introduction

A fundamental challenge for string theory is to explain the cosmology of our Universe. How can the theory describe, or even better predict, basic features of our Universe? Despite considerable effort over the last few years (see [1] – [9] for a partial list of references), still a concrete string theoretic framework for studying cosmology is lacking. The purpose of this article is to report some progress toward this direction by exhibiting new, physically relevant cosmological solutions of superstring theory. These solutions were obtained and analyzed recently in [10], after taking into account thermal and quantum corrections in superstring models for which supersymmetry is spontaneously broken.

At the level of classical string compactifications (with or without fluxes), it seems difficult to obtain realistic, tractable cosmological solutions. In most cases, the classical ground states correspond to static Anti-de Sitter or flat backgrounds and not to cosmological ones. But this classical analysis neglects the thermal and quantum corrections, which inevitably must play an important role in any attempt to identify non-trivial cosmological states.

It is precisely this direction that we wish to explore in this article. It involves studying cosmologies that are generated dynamically at the quantum level of string theory [6–8, 10]. For certain cases the quantum and thermal corrections are under control due to the very special structure of the underlying effective supergravity theory in its spontaneously broken supersymmetric phase.

In order to see how cosmological solutions emerge naturally in this context, consider the case of an initially supersymmetric flat string background at finite temperature. The thermal fluctuations produce a calculable energy density whose back-reaction on the space-time metric and on certain moduli fields gives rise to a cosmological evolution. For temperatures below the Hagedorn temperature, the evolution of the universe is known to be radiation dominated [11, 12].

More interesting cases are those where space-time supersymmetry is spontaneously broken at the string level via freely acting orbifolds [13]– [18]. In these cases, the thermal and supersymmetry breaking couplings correspond to a generalization of Scherk-Schwarz compactification in superstrings. The thermal corrections are implemented by introducing a coupling of the space-time fermion number Q_F to the string momentum and winding num-

bers associated to the Euclidean time cycle S_T^1 . The breaking of supersymmetry is generated by a similar coupling of an internal R -symmetry charge Q_R to the momentum and winding numbers associated to an internal spatial cycle S_M^1 , e.g. the X_5 coordinate cycle.

Two special mass scales appear both associated with the breaking of supersymmetry: the temperature scale $T \sim 1/(2\pi R_0)$ and the supersymmetry breaking scale $M \sim 1/(2\pi R_5)$, where R_0 and R_5 are the radii of the Euclidean time cycle, S_T^1 , and of the internal spatial cycle, S_M^1 , respectively. The initially degenerate mass levels of bosons and fermions split by an amount proportional to T or M , according to the charges Q_F and Q_R . This mass splitting gives rise to a non-trivial free energy density, which incorporates simultaneously the thermal corrections and quantum corrections due to the supersymmetry breaking boundary conditions along the spatial cycle S_M^1 . The back-reaction on the initially flat space-time metric results in different kinds of cosmological evolutions, depending on the initial amount of supersymmetry ($N = 4, N = 2, N = 1$).

In [10] we concentrated on four dimensional heterotic models with initial $N = 4$ and $N = 2$ amount of supersymmetry, leaving the phenomenologically more interesting $N = 1$ cases for future work. Below we summarize some of our main results.

2 Thermal and quantum corrections in heterotic backgrounds

We study the class of four dimensional string backgrounds obtained by toroidal compactification of the heterotic string on T^6 and T^6/\mathbb{Z}_2 orbifolds. The initial amount of space-time supersymmetry is $N_4 = 4$ for the T^6 models and $N_4 = 2$ for the orbifold models. Space-time supersymmetry is then spontaneously broken by introducing Scherk-Schwarz boundary conditions on an internal spatial cycle and by thermal corrections.

The four dimensional one-loop effective action in string frame is given by

$$S = \int d^4x \sqrt{-\det g} \left(e^{-2\phi} \left(\frac{1}{2} R + 2\partial_\mu \phi \partial^\mu \phi + \dots \right) - \mathcal{V}_{\text{String}} \right), \quad (2.1)$$

where ϕ is the $4d$ dilaton field. The ellipses stand for the kinetic terms of other moduli fields. At zero temperature, the effective potential $\mathcal{V}_{\text{String}}$ is given in terms of the one-loop

Euclidean string partition function as follows:

$$\frac{Z}{V_4} = -\mathcal{V}_{\text{String}} \quad (2.2)$$

with V_4 the $4d$ Euclidean volume. At finite temperature, the one-loop Euclidean partition function determines the free energy density and pressure:

$$\frac{Z}{V_4} = -\mathcal{F}_{\text{String}} = P_{\text{String}}. \quad (2.3)$$

In order to determine the back-reaction on the metric and on certain moduli fields, it is convenient to work in the Einstein frame. For this purpose, we define the complex field S , $S = e^{-2\phi} + i\chi$, where χ is the axion field. Then after the Einstein rescaling of the metric, the one loop effective action becomes:

$$S = \int d^4x \sqrt{-\det g} \left[\frac{1}{2} R - g^{\mu\nu} K_{I\bar{J}} \partial_\mu \Phi_I \partial_\nu \bar{\Phi}_{\bar{J}} - \frac{1}{s^2} \mathcal{V}_{\text{String}}(\Phi_I, \bar{\Phi}_{\bar{I}}) \right]. \quad (2.4)$$

Here $K_{i\bar{j}}$ is the metric on the scalar field manifold $\{\Phi_I\}$, parameterized by various compactification moduli and the field S . This manifold includes also the main moduli fields $T_I, U_I, I = 1, 2, 3$, which are the volume and complex structure moduli of the three internal 2-cycles respectively.

In the Einstein frame the effective potential is rescaled by a factor $1/s^2$, where $s = \text{Re}S = e^{-2\phi}$. We have $\mathcal{V}_{\text{Ein}} = \mathcal{V}_{\text{String}}/s^2$. We always work in gravitational mass units, with $M_G = \frac{1}{\sqrt{8\pi G_N}} = 2.4 \times 10^{18} \text{ GeV}$.

What will be crucial in our analysis are some fundamental scaling properties of \mathcal{V}_{Ein} in the limit of large $R_0, R_5 \gg 1$. In this limit, only the temperature scale $T \sim 1/\sqrt{s}R_0$ and three of the main moduli fields, $\{S, T_1, U_1\}$ appear in \mathcal{V}_{Ein} . All other moduli appear in exponentially suppressed contributions:

$$\mathcal{V}_{\text{Ein}} \simeq \frac{F\left(\frac{sR_0^2}{sR_5^2}, \dots\right)}{(st_1u_1)^2} + \mathcal{O}(e^{-c_0R_0-c_5R_5}), \quad (2.5)$$

where the function F will be determined later on. Freezing all other moduli, the classical Kähler potential takes a no-scale structure [19], as was expected from the effective field theory approach:

$$K = -\log (S + \bar{S}) - \log (T_1 + \bar{T}_1) - \log (U_1 + \bar{U}_1) \equiv -3 \log (Z + \bar{Z}), \quad (2.6)$$

with $z = \text{Re } Z$ and $z^3 = st_1 u_1$.

The classical superpotential is constant, and so the gravitino mass scale is given by

$$M^2 = 8e^K = \frac{1}{st_1 u_1} = \frac{1}{z^3}. \quad (2.7)$$

Freezing further $\text{Im } Z$ and defining the field Φ by

$$e^{2\alpha\Phi} = M^2 = \frac{8}{(Z + \bar{Z})^3}, \quad (2.8)$$

we obtain the following kinetic term

$$-g_{\mu\nu} 3 \frac{\partial_\mu Z \partial_\nu \bar{Z}}{(Z + \bar{Z})^2} = -g_{\mu\nu} \frac{\alpha^2}{3} \partial_\mu \Phi \partial_\nu \Phi \quad (2.9)$$

from the Kähler potential. The choice $\alpha^2 = 3/2$ normalizes canonically the kinetic term of the no-scale modulus Φ .

In all, the effective potential in Einstein frame acquires the following structure:

$$\mathcal{V}_{\text{Ein}} \simeq M^4 F\left(\frac{sR_0^2}{sR_5^2}, \dots\right) \simeq M^4 F\left(\frac{M^2}{T^2}, \frac{m_Y^2}{T^2}\right). \quad (2.10)$$

The possible dependence on other Susy mass scales M_Y^2 will become clear latter on, when we consider explicit examples.

3 Thermal and spontaneous breaking of supersymmetry

We first consider the case of a heterotic string background with maximal space-time supersymmetry ($N_4 = 4$). All nine spatial directions as well as the Euclidean time are compactified on a ten dimensional torus. At zero temperature and in the absence of Susy breaking couplings, the Euclidean string partition function is zero due to space-time supersymmetry.

At finite temperature and in the presence of a Scherk-Schwarz Susy breaking coupling, the result is a well defined finite quantity [16]- [18]. At genus one the string partition function is given by:

$$\begin{aligned} Z = & \int_F \frac{d\tau d\bar{\tau}}{4\text{Im}\tau} \frac{1}{2} \sum_{a,b} \frac{(-)^{a+b+ab} \theta \left[\begin{smallmatrix} a \\ b \end{smallmatrix} \right]^4}{\eta(\tau)^{12} \bar{\eta}(\bar{\tau})^{24}} \Gamma_{(5,21)}(R_I) \Gamma_{(3,3)}(R) \\ & \times \sum_{h_0, g_0} \Gamma \left[\begin{smallmatrix} h_0 \\ g_0 \end{smallmatrix} \right] (R_0) (-)^{g_0 a + h_0 b + g_0 h_0} \sum_{h_5, g_5} \Gamma \left[\begin{smallmatrix} h_5 \\ g_5 \end{smallmatrix} \right] (R_5) (-)^{g_5 a + h_5 b + g_5 h_5}. \end{aligned} \quad (3.1)$$

The non-vanishing of the partition function is due to the non-trivial coupling of the $\Gamma(R_0)$ and the $\Gamma(R_5)$ shifted lattices to the spin structures (a, b) . Here, the argument a is zero for space-time bosons and one for space-time fermions. The shifted lattices are given by $\Gamma_{(1,1)} \left[\begin{smallmatrix} h \\ g \end{smallmatrix} \right] (R) = \sum_{m,n} R (\text{Im}\tau)^{-\frac{1}{2}} e^{-\pi R^2 \frac{|2m+g+(2n+h)\tau|^2}{\text{Im}\tau}}$. We are interested in the case for which the radii of three spatial directions are very large, $R_x = R_y = R_z \equiv R \gg 1$, so that the three dimensional spatial volume factorizes $\Gamma_{(3,3)} \cong R^3 \text{Im}\tau^{-\frac{3}{2}} = (V_3/(2\pi)^3) \text{Im}\tau^{-\frac{3}{2}}$.

Before we proceed, the following comments are in order:

- The sector $(h_0, g_0) = (h_5, g_5) = (0, 0)$ gives zero contribution due to the fact that we started with a supersymmetric background.
- In the odd winding sectors, $h_0 = 1$ and/or $h_5 = 1$, the partition function diverges when R_0 and/or R_5 are between the Hagedorn radius $R_H = (\sqrt{2} + 1)/2$ and its dual $1/R_H$: $\frac{1}{R_H} < R_{0,5} < R_H$. The divergence is due to winding states that become tachyonic. Their condensation drives the system towards a phase transition [16]- [18].
- In the regime $R_0, R_5 \gg 1$, there are no tachyons. As we will see, the odd winding sectors as well as the string oscillator states give exponentially suppressed contributions to the partition function. The contributions of the internal $\Gamma_{(5,21)}(R_I)$ lattice states are also exponentially suppressed, provided that the moduli R_I are of order unity.

Thus for large R_0, R_5 , only sectors for which $h_0 = h_5 = 0$ contribute significantly. By utilizing Jacobi identities involving the theta functions, we can see that when $h_0 = h_5 = 0$, we get a non-zero contribution only if $g_0 + g_5 = 1$.

Next, using the relation $\Gamma(R) = \Gamma \left[\begin{smallmatrix} 0 \\ 0 \end{smallmatrix} \right] + \Gamma \left[\begin{smallmatrix} 0 \\ 1 \end{smallmatrix} \right] + \Gamma \left[\begin{smallmatrix} 1 \\ 0 \end{smallmatrix} \right] + \Gamma \left[\begin{smallmatrix} 1 \\ 1 \end{smallmatrix} \right]$ and neglecting the odd winding sectors, we may replace

$$\Gamma \left[\begin{smallmatrix} 0 \\ 1 \end{smallmatrix} \right] \rightarrow \Gamma(R) - \Gamma \left[\begin{smallmatrix} 0 \\ 0 \end{smallmatrix} \right] = \Gamma(R) - \frac{1}{2}\Gamma(2R) \quad (3.2)$$

in the integral expression for Z . For each lattice term we decompose the contribution in modular orbits: $(m_i, n_i) = (0, 0)$ and $(m_i, n_i) \neq (0, 0)$. For $(m_i, n_i) \neq (0, 0)$, the integration over the fundamental domain is equivalent with the integration over the whole strip but with $n_i = 0$. Notice also that the $(0, 0)$ contribution of $\Gamma(R)$ cancels the one of $\frac{1}{2}\Gamma(2R)$. We are

left with the following integration over the whole strip:

$$Z = \frac{V_5}{(2\pi)^5} \sum_{g_0+g_5=1} \int_{||} \frac{d\tau d\bar{\tau}}{4\text{Im}\tau^{\frac{7}{2}}} \frac{\theta \begin{bmatrix} 1 \\ 0 \end{bmatrix}^4 \Gamma_{(5,21)}(R_I)}{\eta(\tau)^{12} \bar{\eta}(\bar{\tau})^{24}} \sum_{m_0, m_5} e^{-\pi R_0^2 \frac{(2m_0+g_0)^2}{\text{Im}\tau}} e^{-\pi R_5^2 \frac{(2m_5+g_5)^2}{\text{Im}\tau}}. \quad (3.3)$$

The integral over τ_1 imposes the left-right level matching condition. The left-moving part contains the ratio $\theta \begin{bmatrix} 1 \\ 0 \end{bmatrix}^4 / \eta^{12} = 2^4 + \mathcal{O}(e^{-\pi\tau_2})$, which implies that the lowest contribution is at the massless level. Thus after the integration over τ_1 ($\tau_2 \equiv t$), the partition function takes the form

$$Z = 2^4 D_0 \frac{V_5}{(2\pi)^5} \sum_{g_1, g_2} \frac{(1 - (-)^{g_1+g_2})}{2} \int_0^\infty \frac{dt}{2t^{\frac{7}{2}}} \sum_{m_1, m_2} e^{-\pi R_0^2 \frac{(2m_1+g_1)^2}{t}} e^{-\pi R_5^2 \frac{(2m_2+g_2)^2}{t}}, \quad (3.4)$$

up to exponentially suppressed contributions that we drop. The factor $2^4 D_0$ is the multiplicity of the massless level.

Changing the integration variable by setting $t = \pi(R_0^2(2m_1+g_1)^2 + R_5^2(2m_2+g_2)^2) x$, the integral over x can be expressed neatly in terms of Eisenstein functions of order $k = 5/2$:

$$E_k(U) = \sum_{(m,n) \neq (0,0)} \left(\frac{\text{Im } U}{|m + nU|^2} \right)^k. \quad (3.5)$$

The pressure in the Einstein frame can be written as

$$P = \frac{Z}{V_4} = C_T T^4 f_{5/2}(u) + C_V M^4 \frac{f_{5/2}(1/u)}{u}, \quad (3.6)$$

where $u = R_0/R_5 = M/T$, and 2

$$f_k(u) = u^{k-1} \left(\frac{1}{2^k} E_k \left(\frac{iu}{2} \right) - \frac{1}{2^{2k}} E_k(iu) \right). \quad (3.7)$$

Here $C_T = C_V \sim n^*$, where $n^* = 8D_0$ is the number of massless fermion/boson pairs. In this particular model the coefficients C_T and C_V are equal due to the underlying gravitino mass/temperature duality. For fixed u the first term stands for the thermal contribution to the pressure while the second term stands for minus the effective potential.

We conclude this section with some further comments.

- The coefficient C_T is fixed and positive as it is determined by the number of all massless boson/fermion pairs in the initially supersymmetric theory.

- The coefficient C_V will depend on the way the Susy-breaking operator Q_R couples to the left and right movers. In general, $Q_F \neq Q_R$ and the temperature / gravitino mass duality will be broken. Then C_V can be either positive or negative.
- For the T_6/\mathbb{Z}_2 orbifold models with $N = 2$ initial supersymmetry, and with Q_R acting only on the left-movers such that $Q_R \neq Q_F$, the net contribution of the twisted sectors to C_V is negative [10]. The change of sign indicates that in the twisted sectors, the states that become massive are the bosons rather than the fermions.

3.1 Small mass scales from Wilson line deformations

A generic supersymmetric heterotic background may contain in its spectrum massive supermultiplets whose mass is obtained by switching on non-trivial continuous Wilson-lines [20]. This is a stringy realization of the Higgs mechanism, breaking spontaneously the initial gauge group to smaller subgroups.

We restrict to arbitrary and small Wilson line deformations starting from a given supersymmetric background where R_I , $I = 6, 7, \dots, 10$ are of the order the string scale. In the zero winding sector, a Wilson line just modifies the Kaluza-Klein momenta, and the corresponding Kaluza-Klein mass becomes

$$\frac{m_I^2}{R_I^2} \longrightarrow \frac{(m_I + y_I^a Q_a)^2}{R_I^2}, \quad (3.8)$$

where Q_a is the charge operator associated to the Wilson-line y_I^a . We can distinguish two different cases: $I = 5$ where R_5 is large, and $I = 6, \dots, 10$ where the R_I are of order the string scale.

Here we shall consider the second case $I = 6, 7, \dots, 10$. In this case, we can set the momentum and winding numbers to zero, $m_I = n_I = 0$, so that the relevant modification in the partition function is the insertion of the term:

$$e^{-\pi t \left(\frac{y_I^a Q_a}{R_I} \right)^2} \simeq 1 - \pi t \left(\frac{y_I^a Q_a}{R_I} \right)^2. \quad (3.9)$$

Then incorporating the effects of the Wilson lines up to quadratic order, we get for the overall pressure:

$$P = C_T T^4 f_{\frac{5}{2}}(u) - D_T T^2 M_Y^2 f_{\frac{3}{2}}(u) + C_V M^4 \frac{f_{\frac{5}{2}}(1/u)}{u} - D_V M^2 M_Y^2 \frac{f_{\frac{3}{2}}(1/u)}{u}. \quad (3.10)$$

Here, $M_Y \sim y_I^a Q_a / R_I$ introduces a new mass scale in the theory, which is qualitatively different than T and M . M_Y is a supersymmetric mass scale rather than a Susy-breaking scale like T and M .

3.2 Scaling properties of the thermal effective potential

The final expression for P contains three mass scales: M , T and M_Y . The first identity it satisfies follows from its definition:

$$\left(T \frac{\partial}{\partial T} + M \frac{\partial}{\partial M} + M_Y \frac{\partial}{\partial M_Y} \right) P = 4P, \quad (3.11)$$

which can be best seen by writing P as

$$P \equiv T^4 p_4(u) + T^2 M_Y^2 p_2(u) = P_4 + P_2, \quad u = \frac{M}{T}. \quad (3.12)$$

Using standard thermodynamic identities, we can obtain the energy density $\rho = \rho_4 + \rho_2$:

$$\rho \equiv T \frac{\partial}{\partial T} P - P = \rho_4 + \rho_2 \quad (3.13)$$

with

$$\rho_4 = \left(3P_4 - u \frac{\partial}{\partial u} P_4 \right) \quad \rho_2 = \left(P_2 - u \frac{\partial}{\partial u} P_2 \right). \quad (3.14)$$

In the sequel, we allow the Susy-breaking scales T and M to vary with time while fixing the supersymmetric mass scale M_Y and also u , and investigate the back-reaction to the initially flat metric and moduli fields.

4 Gravitational equations and critical solution

We assume that the back-reacted space-time metric is homogeneous and isotropic

$$ds^2 = -dt^2 + a(t)^2 d\Omega_k^2, \quad H = \left(\frac{\dot{a}}{a} \right), \quad (4.1)$$

where Ω_k denotes the three dimensional space with constant curvature k and H is the Hubble parameter.

From the fact that $-P$ plays the role of the effective potential and the relation between the gravitino mass scale M and the no scale modulus Φ , $M = e^{\alpha\Phi}$, we obtain the field equation:

$$\ddot{\Phi} + 3H\dot{\Phi} = \frac{\partial P}{\partial \Phi} = \alpha u \left(\frac{\partial P}{\partial u} \right)_T = -\alpha (\rho_4 - 3P_4 + \rho_2 - P_2). \quad (4.2)$$

The remaining equations are the gravitational field equations. These are the Friedmann-Hubble equation,

$$3H^2 = \frac{1}{2}\dot{\Phi}^2 + \rho - \frac{3k}{a^2}, \quad (4.3)$$

and the equation that follows from varying with respect to the spatial components of the metric:

$$2\dot{H} + 3H^2 = -\frac{k}{a^2} - P - \frac{1}{2}\dot{\Phi}^2. \quad (4.4)$$

This last equation can be replaced by the linear sum of the two gravitational field equations, so that the kinetic term of Φ drops out:

$$\dot{H} + 3H^2 = -\frac{2k}{a^2} + \frac{1}{2}(\rho - P). \quad (4.5)$$

4.1 Critical solution

The scaling properties of the thermal effective potential suggest to search for a solution where all varying mass scales of the system, $M(\Phi)$, T and $1/a$, remain proportional during time evolution:

$$e^{\alpha\Phi} \equiv M(\Phi) = \frac{1}{\gamma a} \implies H = -\alpha\dot{\Phi}, \quad M(\Phi) = uT \quad (4.6)$$

with γ and u fixed in time. Our aim is thus to determine the constants γ and u .

Along the critical trajectory, the compatibility of the Φ -equation of motion with the gravitational field equations requires that

$$r_4 = \frac{6\alpha^2 - 1}{2\alpha^2 - 1}p_4, \quad \left(r_4 = 4p_4 \quad \text{for } \alpha^2 = \frac{3}{2} \right), \quad (4.7)$$

$$-2k\gamma^2 = \frac{2\alpha^2 - 1}{2} \frac{r_2 - p_2}{u^2} M_Y^2, \quad \left(-2k\gamma^2 = \frac{(r_2 - p_2)}{u^2} M_Y^2 \quad \text{for } \alpha^2 = \frac{3}{2} \right), \quad (4.8)$$

where $r_4 = \rho_4/T^4$ and $r_2 = \rho_2/(T^2 M_Y^2)$. The first equation is an algebraic equation for the complex structure-like ratio u . The second equation determines the spatial curvature of the solution.

Having solved for the compatibility equations, the dynamics for the scale factor a is governed by an effective Friedmann-Hubble equation as follows:

$$3H^2 = -\frac{3\hat{k}}{a^2} + \frac{c_r}{a^4}, \quad (4.9)$$

where

$$3\hat{k} = -\frac{1}{\gamma^2} \frac{6\alpha^2}{6\alpha^2-1} \frac{1}{u^2} \left(\frac{3(2\alpha^2-1)}{4}(r_2-p_2) + r_2 \right) M_Y^2, \quad (4.10)$$

$$c_r = \frac{1}{\gamma^4} \frac{6\alpha^2}{6\alpha^2-1} \frac{r_4}{u^4} = \frac{1}{\gamma^4} \frac{6\alpha^2}{2\alpha^2-1} \frac{p_4}{u^4}. \quad (4.11)$$

The following comments are in order:

- Clearly, a necessary condition for the curvature \hat{k} not to vanish is to have non trivial Wilson lines in any of the directions 6, 7, 8, 9, 10. Models with both positive and negative \hat{k} can be constructed [10].
- The value of the ratio $u = M/T$ was obtained by solving the compatibility equations numerically. It can be large or small depending on the model. In other words there are models with a hierarchy for the Susy-breaking scales M and T . In all models considered in [10], the value for the effective coefficient c_r was positive.

5 Concluding remark

The purpose of this talk is to emphasize the plausible existence of cosmological superstring solutions, inflationary or not, which are generated dynamically at the quantum string level. Such cosmologies arise naturally from an initially flat spacetime, once supersymmetry is spontaneously broken by thermal and quantum effects. They are examples of no-scale, radiatively induced cosmologies. We believe that this new set-up will result in a coherent and fruitful framework in order to understand superstring cosmology.

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